In this short note, we explain why the distance between the point $P = (s, t, u)$ and the plane $ax + by + cz + d = 0$ is

$$\frac{|as + bt + cu + d|}{\sqrt{a^2 + b^2 + c^2}}.$$

In the lecture notes, it is explained why this distance should be $|\text{proj}_{\vec{n}} \overrightarrow{QP}| = |\text{comp}_{\vec{n}} \overrightarrow{QP}|$, where Q is any point on the plane, and $\vec{n} = \langle a, b, c \rangle$ is a normal vector.

Since this is $\dfrac{|\overrightarrow{QP} \cdot \vec{n}|}{|\vec{n}|} = \dfrac{|\overrightarrow{QP} \cdot \vec{n}|}{\sqrt{a^2 + b^2 + c^2}}$, it is left to show why $|\overrightarrow{QP} \cdot \vec{n}| = |as + bt + cu + d|$. Let's say $Q = (s', t', u')$. Then since Q is on the plane, $as' + bt' + cu' + d = 0$, or $as' + bt' + cu' = -d$. Note also $\overrightarrow{QP} = \langle s - s', t - t', u - u' \rangle$. So $\overrightarrow{QP} \cdot \vec{n} = a(s - s') + b(t - t') + c(u - u')$
$= as + bt + cu - (as' + bt' + cu') = as + bt + cu + d$.